



Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



MATHEMATICS

9709/22

Paper 2 Pure Mathematics 2

October/November 2021

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

- 2 (a) Sketch, on the same diagram, the graphs of $y = x + 3$ and $y = |2x - 1|$. [2]

- (b) Solve the equation $x + 3 = |2x - 1|$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Find the value of y such that $5^{\frac{1}{2}y} + 3 = |2 \times 5^{\frac{1}{2}y} - 1|$. Give your answer correct to 3 significant figures. [2]

.....

.....

.....

.....

.....

.....

3 The curve with equation

$$y = 5x - 2 \tan 2x$$

has exactly one stationary point in the interval $0 \leq x < \frac{1}{4}\pi$.

Find the coordinates of this stationary point, giving each coordinate correct to 3 significant figures.

[6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

4 Given that $\int_a^{a+14} \frac{1}{3x} dx = \ln 2$, find the value of the positive constant a . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

5 A curve has equation $x^2 + 4x \cos 3y = 6$.

Find the exact value of the gradient of the normal to the curve at the point $(\sqrt{2}, \frac{1}{12}\pi)$. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6 (a) By sketching a suitable pair of graphs on the same diagram, show that the equation

$$\ln x = 2e^{-x}$$

has exactly one root.

[2]

(b) Verify by calculation that the root lies between 1.5 and 1.6.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = e^{2e^{-x_n}}$$

converges, then it converges to the root of the equation in part (a). [1]

.....

.....

.....

.....

.....

.....

.....

(d) Use the iterative formula in part (c) to determine the root correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 7 (a) Prove that $4 \sin x \sin(x + \frac{1}{6}\pi) \equiv \sqrt{3} - \sqrt{3} \cos 2x + \sin 2x$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the exact value of $\int_0^{\frac{5}{6}\pi} 4 \sin x \sin(x + \frac{1}{6}\pi) dx$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the smallest positive value of y satisfying the equation

$$4 \sin(2y) \sin\left(2y + \frac{1}{6}\pi\right) = \sqrt{3}.$$

Give your answer in an exact form.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.